

Vertical transports produced by double-diffusive plumes in a confined homogeneous environment

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Double-diffusive plumes, injected into surroundings of nearly the same density, are observed to separate near the source and to produce both upward and downward convection, which is enhanced as layers form at the top and bottom of the experimental tank. This process has been quantified in this paper for both sugar plumes in homogeneous salt solution and salt plumes in sugar solution, over a range of small density differences on either side of zero. The tank was fitted with a partition at mid-depth, which could be closed at the end of a known period of input, so that the concentration changes above and below the source could be deduced. The density of the input and tank fluids, and that of the upper and lower layers, was measured in each case, as well as a second property, conductivity for the salt and optical rotation for the sugar inputs. These measurements were motivated by, and shed light on, the experimental results of Turner & Veronis (2000). There are large differences between the upward and downward transports produced by the two types of plume, because of the different rates of diffusion of salt and sugar in and out of the plumes and the subsequent transports through the interfaces that form as the plumes spread out along the top and bottom boundaries. One result is that at a density ratio close to 1 a salt plume produces equal upward and downward transports of salt, whereas at the same density ratio a sugar plume leads to a much larger downward flux of sugar. This is consistent with the ‘diffusive’ final state observed by Turner & Veronis (2000), and this conclusion is supported by a more detailed analysis of the redistribution of the tank fluid.

1. Introduction

Many phenomena observed in the ocean are the result of the interaction between several spatially separated sources of water having different temperatures and salinities. Some of these sources are relatively small and localized; the bottom water of all the world’s oceans is known to form in just a few limited regions, for example Antarctic Bottom Water in the Weddell Sea. The relatively cold, fresh and dense downflow that forms in that region can be regarded as a plume that mixes with its surroundings before spreading northwards at the bottom. Another example of a source that is essentially double-diffusive in character, because of the opposing effects of temperature and salinity on the density, is the influx of anomalously warm Atlantic water into the Arctic Ocean reported by Carmack *et al.* (1997). This takes the form of persistent multiple intrusions 40–60 m thick at depths below 200 m, extending laterally in a coherent manner through the Atlantic water and the upper deep waters of the Arctic. These authors noted that the observed layers can support both diffusive and finger convection, and suggested that they are self-organizing and driven by these double-diffusive processes.

Turner & Veronis (2000, herein after referred to as TV) have carried out a series of experiments aimed at exploring two-dimensional double-diffusive processes driven by horizontal and vertical gradients of temperature and salinity in the ocean. They used the salt–sugar analogue system, with plumes of salt and sugar solutions supplied at opposite ends of a tank 750 mm long filled with initially homogeneous fluid, and an overflow tube at the centre to keep the volume constant. Various input conditions and geometries were used, but the comparison between three related experiments was particularly interesting. In these, the fluid depth was 150 mm, both the inputs and the withdrawal tube were placed at mid-depth, and the tank fluid and the salt and sugar supplies had close to the same density, 1.100 g cm^{-3} . The only difference between the three runs was that the initial composition of the solution in the tank was, respectively, pure salt, pure sugar or a 50:50 mixture of the two. Whenever the composition of the input and the tank solution into which it flowed was different, strong vertical convection was observed, and plume fluid was evidently being transported upwards and downwards.

After about 100 h of continuous injection of salt and sugar, each at a rate of 5.0 ml min^{-1} , all three experiments tended to the same asymptotic distributions of density, salt and sugar. The development of a strong vertical stratification of these properties is the most striking consequence of double diffusion, and this will also be significant in the oceanic heat–salt system. With two salt sources in a tank of salt solution the total density range must always lie within that of the inputs, and when all three densities are equal, no density difference could develop at all. In the double-diffusive system studied, however, the overall asymptotic density difference was 0.044 g cm^{-3} . The separate contributions of salt and sugar to the densities at the top and bottom were also distinctive. The upper and lower layers were separated by a sharp central interface (because of the withdrawal at that level) and were weakly stratified. This final state corresponded to the one-dimensional ‘rundown’ of a layer of salt solution above sugar solution, with a slightly higher unstable concentration of salt in the top layer compared to the bottom and a very stable sugar distribution (i.e. a much larger sugar concentration in the bottom layer than in the top one). However, these distributions were, in fact, achieved through vertical convection in the plumes, combined with slow advection towards the sink at mid-depth, and followed by the action of many intrusions, which remained active in the dynamic final state and maintained a strong communication between the two ends of the tank.

The detailed understanding of the observations described above is a major objective of the present paper. Another clue, and a second motivation for the experiments described here, is provided by the comparison between the TV runs in which first salt and then sugar was the initial homogeneous solution in the tank. It was observed that, though the asymptotic behaviour was the same in the two cases, the rate of change of the vertical density difference in the early stages of the experiments (i.e. the first 10 h) was very different. Double-diffusive convection was prominent first near the sugar source with a homogeneous salt solution in the tank, and near the salt source in a tank of sugar, and in the latter case a thin surface layer with a diffusive interface below it developed rapidly. TV suggested that the key to this difference, and also to the later evolution towards the final state, should be sought in the differences between the two types of input plume, sugar feeding into salt solution and salt into sugar.

The experiments described in this paper, conducted in a finite tank, are relevant for explaining the TV results, since in both cases the density structure is developed by the confining effect of the top and bottom boundaries. Three separate, but linked, double-diffusive processes can be identified in the TV experiments. First, there is the

separation of the initial plume, with convection both upwards and downwards near the source. These plumes rapidly reach the top and bottom boundaries, and further convection occurs through transports across the interfaces that form, producing a vertical density gradient locally. Finally, on a longer time scale, intrusions develop and spread out along the tank. The first two processes are those that are observed and studied here. The smaller horizontal dimension compared to that used by TV will change the time scale of the evolution, but this will have a less significant effect on the vertical property gradients.

The above introductory remarks have concentrated on the unanswered questions in a previous paper that have provided the immediate motivation for the present work. Other observations and measurements of the plumes themselves will now be referred to briefly. The upward and downward convection produced by the splitting of a plume of sugar flowing into homogeneous salt solution was first described and photographed by Turner & Chen (1974), and since then the phenomenon has been observed and commented on by many experimenters using both the salt–sugar and heat–salt systems. In discussions of the Turner & Chen and subsequent experiments it has been asserted that the strongest, nearly symmetrical, vertical convection is observed when the densities of the plume and the tank fluid are the same; but this statement is at best semi-quantitative (it has been based on measurements of density using a hydrometer) and it has not been tested in detail. TV also repeated this remark, but the need for more precise measurements, covering a range of small density differences, became apparent in exploratory experiments that led to the series presented here.

McDougall (1983) has carried out experiments on double-diffusive plumes, both in the heat/salt and salt/sugar systems, that are closely related to the measurements to be described in this paper. He identified four cases for the heat/salt system, in which lighter or denser warm salty water flows into cool fresh water and the reverse, where lighter or denser cool fresh water flows into warm salty water. His discussion of the density changes in the plumes and the environment assumed that only heat is transferred, and that the exchange of salt is insignificant. His experiments (conducted in a tank with a partition, the technique adopted in the present study) used only warm salty inputs since he argued that the cold fresh plumes would be dynamically the same. The analogue system corresponding to his experiments with warm salty plumes is a sugar plume flowing into salt solution. McDougall (1983) addressed both the early stages of the separating flow (but only for the warm salty plumes) and also the later evolution in both systems. At longer times, he showed that the ‘filling box’ mechanism (Baines & Turner 1969) combined with further vertical transports across double-diffusive interfaces in a confined environment, certainly changes the density structure and affects the motion of the plumes.

The experiments described below cover all four cases using the salt/sugar analogue (light and dense sugar and salt plumes feeding into a homogeneous solution of the other property), and in this system it seems likely that the diffusion of both solutes could be significant. The ‘filling box’ process has been found to be important at all stages of the present experiments, and it dominated the measurements at the larger density differences. Turner (2001) gives a preliminary report on some of the results discussed here.

2. Experimental method and data analysis

The new experiments were carried out in a Perspex tank 300 mm × 150 mm in cross-section and 350 mm deep, which is shown schematically in figure 1. At 150 mm

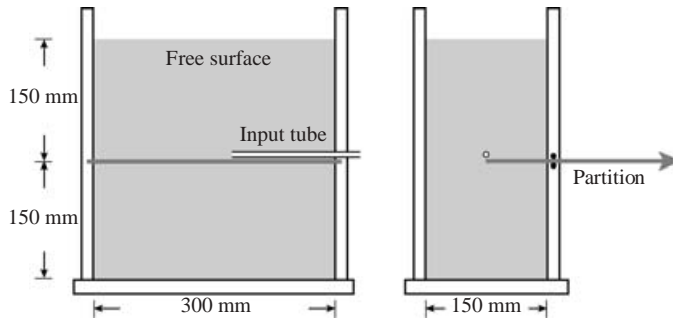


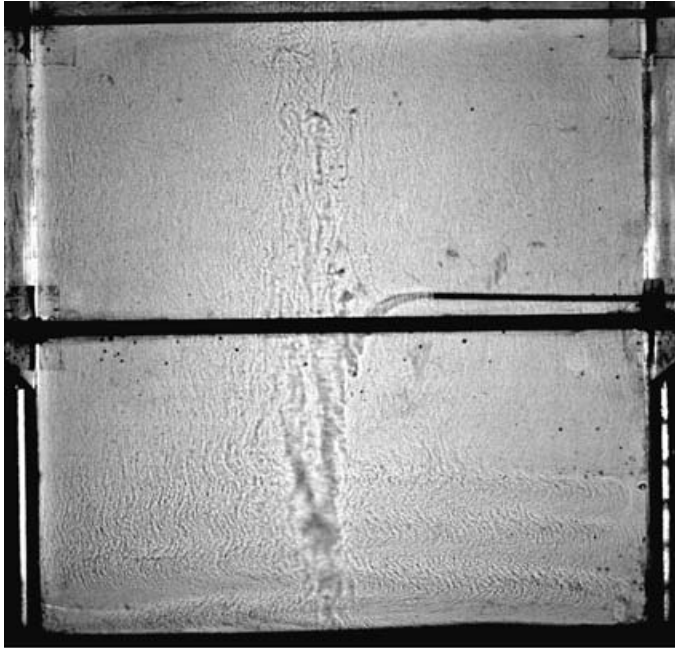
FIGURE 1. Sketch of the experimental tank, showing the input tube and the sliding partition that allowed the upper and lower layers to be isolated and sampled.

above the base, the tank was fitted with a partition, sliding through a waterproof slot in the front (300 mm) wall and in grooves in the side (150 mm) and back walls. This partition was left open during the period of a run, then closed to isolate the upper and lower compartments. For each run, the tank was filled to a depth of 300 mm with either salt or sugar solution, of specific gravity close to 1.100, measured using an Anton Paar densitometer (to 1 part in 10^5). The input solution of the other solute was prepared, with its density varied systematically on either side of the density of the tank fluid. This was injected, using a peristaltic pump, through a hypodermic tube (3 mm o.d., 2 mm i.d.) inserted through the sidewall 10 mm above the partition. The fact that the input was not precisely at mid-depth did not affect the measurements or their interpretation.

Two typical experiments will be described first. Figure 2(a) shows a shadowgraph photograph of a run in which a slightly denser plume of salt solution is being injected into a homogeneous tank of sugar. The plume is separating, producing convection up and down, but the plume is initially depositing salt predominantly at the bottom. This, in turn, produces layering and further vertical convection is seen, extending into the upper layer. Figure 2(b) shows a slightly less dense plume of sugar flowing into a homogeneous tank of salt, with dye added to mark the convection region. These pictures will be discussed in more detail when the corresponding flux measurements are presented.

Four series of experiments were conducted, using constant input rates of 5 ml min^{-1} and 20 ml min^{-1} for both sugar and salt plumes. As it left the input tube, the flow was laminar, the Reynolds numbers at the lower rate being about 25 for sugar plumes, based on the diameter and the input fluid viscosity, and 45 for salt plumes, and four times these values at the higher rate. The flows became turbulent within a few diameters from the nozzle (always a small fraction of the total path of a plume), at a point that depended on both the flow rate and the density difference; these two parameters also determined the trajectory of the plume. The inflow was continued for typically 30 min in experiments using the higher flow rate and 100 min in the experiments at the lower rate, leading to an increase in volume of 500–600 ml (or a depth of 10–12 mm) in each case. This was long enough for a sufficient concentration of the solute in the source to be built up to give an accurate measurement. At the conclusion of a run, the partition was immediately closed and the upper layer stirred and sampled, to provide a measure of the total upward solute transports. This layer was then drained, the partition opened, and the lower layer stirred and sampled to give the corresponding downward transports.

(a)



(b)



FIGURE 2. Shadowgraph pictures of two contrasting experiments, both of which developed vigorous convection upwards and downwards. (a) A salt plume, denser than the sugar solution into which it flowed by 6 parts in 10^4 , 100 min after the input started at 5 ml min^{-1} . Later measurements showed that nearly equal amounts of salt were transported up and down. (b) A sugar plume (lighter by 8 parts in 10^4), flowing into salt solution at 5 ml min^{-1} . Photo taken at 80 min after the start and 2 min after a dye streak was added near the centre. In this case, substantially more sugar was transported down than up, which is not obvious from the visual observations.

The density of the samples was measured in every case, and also optical rotation for the experiments with a sugar input or conductivity for runs with a salt input. This second property was chosen because it was in each case the most sensitive measure of a low concentration of the input solute in the concentrated tank solution. The fractions of the input solute transported up and down was deduced using direct calibrations, based on known dilutions of the input solution with the tank fluid for each run. For the sugar input, the optical rotation was a linear function of the sugar concentration, and virtually independent of the salt concentration. For the salt input, the conductivity was also linear in salt concentration over the low concentration range involved, though it was decreased compared to a simple salt solution by the presence of sugar, to an extent that was accounted for by the individual calibration for each run.

Alternatively, the density contribution of the input solute to that of each layer, and hence the fractions transported up and down, can be calculated using two measured properties and the inversion procedure developed by Ruddick & Shirtcliffe (1979). This was, in fact, done for all runs in the concentration ranges covered by their inversion, since it also makes it possible to deduce how the tank solute (which was initially homogeneous) was redistributed in the tank by the convection in the plumes. The following is the procedure adopted once the separate contributions of salt (αT) and sugar (βS) to the density have been found using the inversion calculation. Assume for example that sugar (S) is the input solute. (When salt (T) is the input solute the procedure is the same, with αT replacing βS wherever it occurs, and vice versa.)

Since the plumes have a finite volume flux, as well as carrying the input solute (S), the volume of fluid in the tank is increased during an experiment. Thus, the sampled volume V_2 above the partition is larger than V_1 below, and the measured concentration must be corrected using the volume ratio before the upward solute transport by plumes can be compared with that in the lower layer. (No solute is transported upwards by advection of the lower layer as a whole, provided the top of the region into which the descending plume is mixed remains below the level of the partition.)

Defining

$$r = \frac{V_2}{V_1}, \quad R = \frac{r\beta S_2}{\beta S_1}, \quad (1)$$

which can be calculated directly from the measured quantities, the fractions of the input solute in the two layers are:

$$\left. \begin{aligned} \text{Fraction in the bottom layer} &= 1/(1 + R), \\ \text{Fraction in the top layer} &= R/(1 + R). \end{aligned} \right\} \quad (2)$$

In the heat/salt system, when a hot salty plume is injected into a tank of fresh water, it is clear that measurements of the increases in temperature and salinity will show directly how the two components have been distributed during an experiment. For the salt/sugar analogue system, it is the redistribution of the tank solute (T) that is equivalent to following a second input property. The upward and downward transports of T can be calculated in essentially the same way as the transport of the input solute (S), once it is recognized that the source solute flux is associated with an input of fresh water and hence with a negative anomaly of the tank solute.

Denote the measured values of the input fluid properties by ρ_i , S_i and T_i , those of the tank fluid by ρ_0 , S_0 and T_0 and the values in the lower and upper layers by subscripts 1 and 2, respectively. Suppose that (as in all the experiments) the initial volumes in the upper and lower layers are equal ($=V_1$) and that the (measured) total volume V_i of source fluid is added during an experiment. Then the mass conservation

equation is

$$V_i \rho_i + 2V_1 \rho_0 = V_1 \rho_1 + V_2 \rho_2. \quad (3)$$

The corresponding equations for the solutes are

$$V_i \rho_i S_i + 2V_1 \rho_0 S_0 = V_1 \rho_1 S_1 + V_2 \rho_2 S_2, \quad (4)$$

$$V_i \rho_i T_i + 2V_1 \rho_0 T_0 = V_1 \rho_1 T_1 + V_2 \rho_2 T_2. \quad (5)$$

Using (4) and (5) with (3), the conservation equations for the solute anomalies from the initial values in the tank are

$$V_i \rho_i (S_i - S_0) = V_1 \rho_1 (S_1 - S_0) + V_2 \rho_2 (S_2 - S_0), \quad (6)$$

$$V_i \rho_i (T_i - T_0) = V_1 \rho_1 (T_1 - T_0) + V_2 \rho_2 (T_2 - T_0). \quad (7)$$

The terms on the right-hand side of (6) are those that have already been used in (1) and (2) to find the distribution of the input solute S . More care is needed in the calculations for the tank solute T . The terms on the right-hand side of (7) are less precisely known since they involve small differences between large concentrations that are not so accurately determined by the inversion procedure. There are two alternative methods of using the measurements to calculate a value of T_0 to insert in (7). First, the input term in (5) is zero when T is the tank fluid, so T_0 can be calculated since all the other quantities are known. Secondly, T_0 can be obtained directly from the measured ρ_0 , using the polynomial expression derived by Ruddick & Shirtcliffe (1979).

For most runs, the two anomaly values calculated in this way were not very different, and the mean of the two was used in plotting the data. However, in a few cases the differences were larger, indicative of larger errors in one or more T and S estimates. For these runs, the knowledge of the input parameters on the left-hand sides of (6) and (7) made it possible to adjust T_1 and T_2 to achieve consistency. In particular, we can use the constraints that in the (T, S) -plane the final mean values in the tank (T_m, S_m) should lie on the straight line joining (T_0, S_0) and (T_i, S_i) and also on the line joining (T_1, S_1) and (T_2, S_2) . The accuracy of the plotted values will be discussed further in the next section.

3. Quantitative results for experiments with an inflow rate of 5.0 ml min^{-1}

3.1. Fraction of the input solute transported up and down

First, we will present the results for the experiments with the lower volume flux in the plumes, 5 ml min^{-1} (the rate used for each source in the TV experiments), concentrating initially on the experiments with density differences close to and on either side of zero. In figure 3, the fractions of the input solute transported up and down, calculated using the inversion procedure and the relations (2), are plotted against the density ratio ρ_i/ρ_0 (input/tank fluid) for both sugar and salt inputs. The curves have been drawn to the points by eye, rather than using a more sophisticated fitting procedure, but it is important to note that they have been made antisymmetric, i.e. the total of the upward and downward fractional transports is constrained to add to unity. The experimental values for the up and down transports at each density ratio therefore always deviate in opposite directions from these curves.

A formal error analysis would be of little value here. For each point, it would need to combine estimated errors for two experimental techniques, followed by the inversion calculation. Instead, the deviation of the individual points from the mean curves can be regarded as a measure of their reliability. Another indication of the accuracy, independent of the inversion procedure, is provided by a comparison with

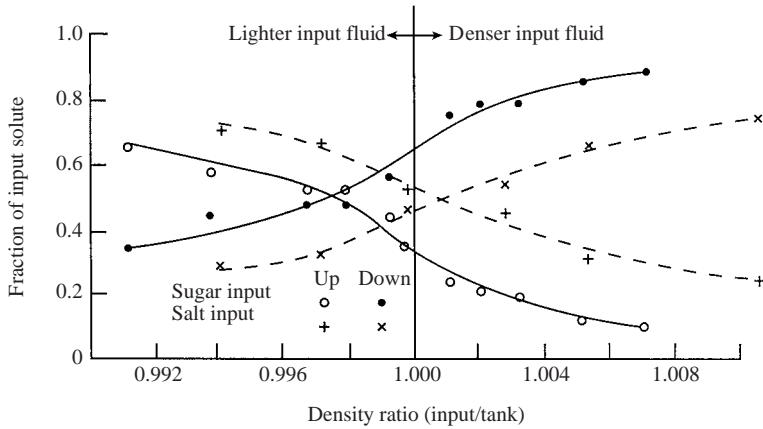


FIGURE 3. The fractions of input solute transported up and down by sugar plumes in a tank of salt solution and salt plumes in a tank of sugar (flow rate 5 ml min^{-1}), over a range of density ratios between the input and tank fluid.

the values derived from the direct calibrations. These will be discussed in §3.4 in the context of measurements at larger density differences, and are plotted as the smaller dots in figure 6, but it is appropriate to refer to them here in the context of error assessment. They differ only marginally from the points obtained using the inversion, though the upward and downward values are determined independently rather than necessarily summing to unity, as imposed by (2).

It is clear that there is a substantial difference in the transports produced by the sugar and salt plumes. There are equal upward and downward fluxes of salt at a density ratio close to unity, the conditions of the experiment shown in figure 2(a). The measurements for a sugar plume with a slightly smaller density ratio (the experiment photographed in figure 2b) show a predominantly downward sugar flux, though this is not at all obvious from the visual observations, and the transports of sugar are equal only for a considerably lighter sugar input. A detailed discussion of these differences will be deferred to a later section.

3.2. Counterbuoyant fluxes of the input solute

Another way of presenting a subset of these same measurements is shown in figure 4. Only the fractional flux of each input solute in the direction opposite to the sense of the initial buoyancy of the plume has been shown, i.e. the downward flux for light plumes and the upward flux for dense plumes. These are now plotted against (the modulus of) the density difference between the tank and the input fluids (rather than the density ratio), so that both light and dense plumes may be more directly compared.

Here again, the difference between sugar and salt plumes is clear, and the contrast as zero density difference is approached is particularly striking. For salt plumes flowing into sugar solution, dense inputs have a slightly larger counterbuoyant transport than light inputs at zero density difference, whereas for sugar plumes the counterbuoyant transport for light inputs is twice as large as it is for dense inputs.

3.3. The fraction of the tank solute transported up and down

Using the methods outlined at the end of §2, the corresponding proportional transports, upward and downward, of the negative solute anomalies of tank solute

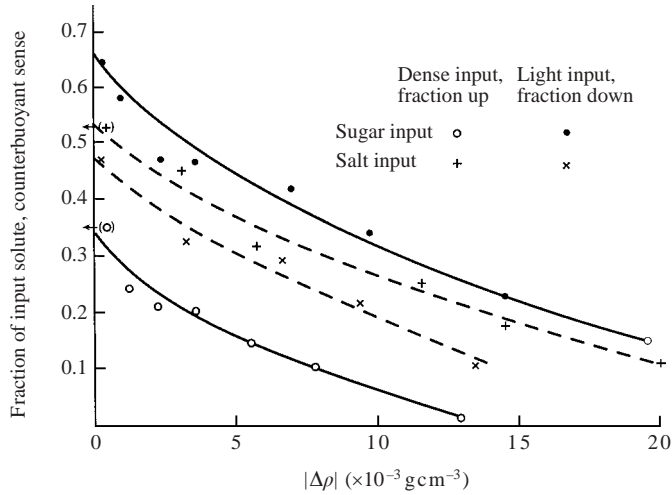


FIGURE 4. Another presentation of a subset of the data in figure 3. Only the fractional solute fluxes in the sense opposite to the initial buoyancy are plotted, as functions of the (modulus of) the density difference between the input and tank solutions. Flow rate 5 ml min^{-1} .

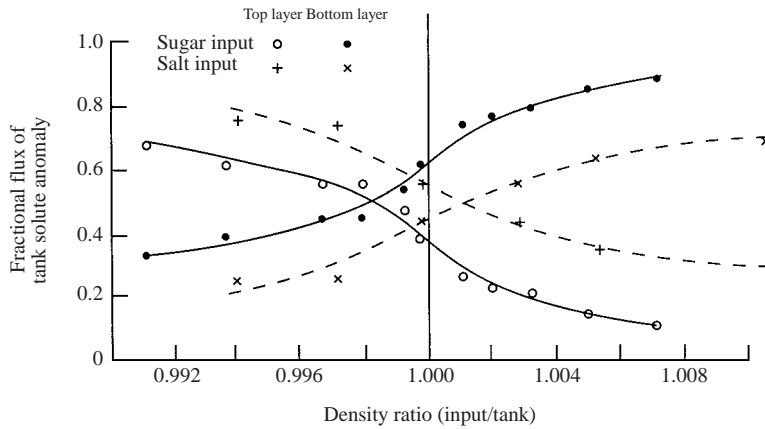


FIGURE 5. The fractions of tank solute anomalies (all of which are negative) transported up and down, produced by convection set up by plumes of salt or sugar flowing at a rate of 5 ml min^{-1} into a homogeneous solution of the other property. The same experiments and data set have been used as for figure 3.

have been calculated for the same data set, i.e. for the experiments with sugar and salt plumes and density ratios close to and on either side of unity. These are plotted in figure 5, using the same symbols as in figure 3. Again, there is a systematic variation with density ratio and a difference between the sugar and salt plumes.

A simple, direct way to compare the effects of sugar and salt plumes with $\rho_i/\rho_0 = 1$ on the vertical distribution of the two solutes is to use the measurements of αT and βS in the top and bottom layers for the two runs having density ratios closest to 1. Run 13, with sugar input and $\rho_i/\rho_0 = 0.99976$, resulted in $\beta S_1 = 0.00474$ (all values are given in units of g cm^{-3}), $\beta S_2 = 0.00244$ with a stabilizing density difference due to sugar of 0.00230 and $\alpha T_1 = 0.09549$, $\alpha T_2 = 0.09732$, implying a destabilizing density difference due to salt of 0.00183. The final distributions in this experiment were therefore in the diffusive sense, with a density ratio $R_\rho = 1.26$. For run 26, with a salt

		Sugar plume	Salt plume
Flow rate 5 ml min ⁻¹	$\beta S'_1/\alpha T'_1$	-1.089 $s = 0.021$	-0.823 $s = 0.059$
	$\beta S'_2/\alpha T'_2$	-0.975 $s = 0.034$	-0.987 $s = 0.044$
Flow rate 20 ml min ⁻¹	$\beta S'_1/\alpha T'_1$	-1.078 $s = 0.032$	-0.864 $s = 0.051$
	$\beta S'_2/\alpha T'_2$	-0.966 $s = 0.058$	-0.968 $s = 0.040$

TABLE 1. Mean ratios of sugar and salt anomaly fluxes $\beta S^1/\alpha T^1$, both down (subscript 1) and up (subscript 2), for all experiments having small values of $\Delta\rho$ on either side of zero. The standard deviations s about the mean are also given.

input and $\rho_i/\rho_0 = 0.99982$, the corresponding measured values are $\beta S_1 = 0.096746$, $\beta S_2 = 0.096233$, which give a stabilizing difference of 0.000513, and $\alpha T_1 = 0.003235$, $\alpha T_2 = 0.003420$, with a destabilizing difference of 0.000185. This is again in the diffusive sense, with a larger density ratio $R_\rho = 2.77$; note too that the destabilizing salt difference is only a tenth of that produced by the sugar plume in about the same time.

What should be a more accurate method of determining the ratios of the anomaly fluxes at density ratios close to 1 is to combine all the measurements of αT and βS in the two layers which were used to prepare figures 3 and 5. The scatter between individual calculated values was found to be large, however, and considerable smoothing is needed to produce consistent results. Eventually, it was decided not to present the individual values or to attempt to document any systematic variation of this flux ratio with the density ratio, but just to calculate separate means for all the light inputs and all the dense inputs over the range of density ratios covered in figures 3 and 5. In the following, a more compact notation will be used for the anomaly variables, replacing $(T_1 - T_0)$ by T_1^1 and so on.

Thus, there are, in principle, eight mean values in all, if we distinguish between the flux ratios for upward and downward solute transports, for sugar and salt plumes, and lighter and denser inputs. All ratios are presented in the form sugar flux divided by salt flux in equivalent density units, i.e. $\beta S^1/\alpha T^1$, regardless of the solute in the plume. The ratios do not, in fact, vary significantly as the density difference changes from positive to negative, as is to be expected if the value at zero density difference is to have an unambiguous meaning, so the results for light and dense plumes have been grouped together. The remaining four mean values for all the experiments with an inflow rate of 5 ml min⁻¹, and the limited range of density differences, are set out in the first section of table 1; all of these are, of course, negative since the anomaly flux of the tank solute is negative.

The largest differences are between the downward sugar/salt flux ratios for the sugar and salt plumes. For sugar plumes, the mean ratio is -1.089 and for salt plumes it is -0.823, indicating that relatively more of the input solute is transported down in each case. All the upward sugar/salt flux ratios are nearly the same; the mean value for all the experiments is -0.98, not significantly different from -1.00. Table 1 also gives the corresponding values of the flux ratios for the experiments with the higher flow rate, to be described in §4.

These ratios will now be used, together with the fraction of the input solute transported up and down at zero density difference (taken from figure 3) to calculate

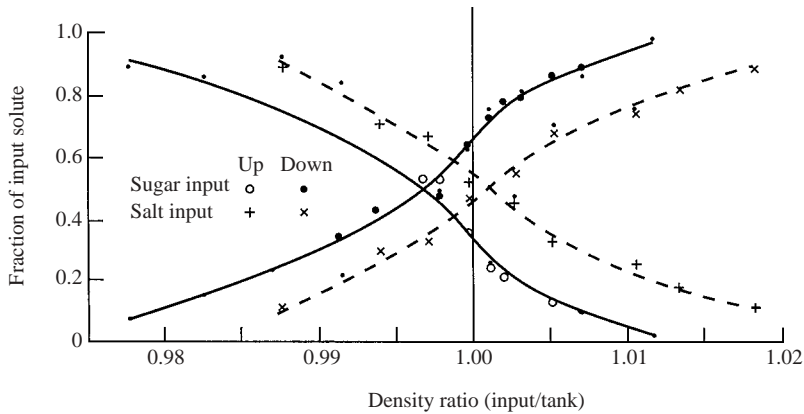


FIGURE 6. The fractions of input solute transported up and down for all the experiments shown in figure 3, plus some extra runs with larger and smaller density ratios to show the asymptotic behaviour.

the vertical solute gradients due to both kinds of plumes. For a sugar plume at a flow rate of 5 ml min^{-1} the fractional upward and downward sugar transports are 0.34 and 0.66, respectively, giving a stabilizing sugar difference of 0.32 per unit sugar input. From the ratios given in table 1, the corresponding changes in salt concentration in the top and bottom layers are -0.349 and -0.606 with a destabilizing salt difference (in the same units) of 0.257. The stratification is indeed diffusive, with a density ratio $R_\rho = 1.25$; the density change at the top is insignificant and at the bottom there is an increase of 0.05. For a salt plume the upward and downward transports are 0.53 and 0.47 with a destabilizing salt difference of 0.06; the corresponding changes in tank sugar concentration are -0.523 and -0.387 and the stabilizing difference is 0.136; the system is diffusive and $R_\rho = 2.27$. There is again a negligible change in density in the top layer and an increase of 0.08 in the bottom. These results confirm those based on just two individual runs, though the initial expectation was that those earlier calculations would be less accurate.

3.4. Measurements with larger values of the density difference

So far, results have been given only for the subset of the experiments closest to $\rho_i/\rho_0 = 1$, the range of most interest in the context of the TV experiments. Further experiments were, however, carried out using larger density differences in both senses, to investigate the asymptotic behaviour as the counterbuoyant fluxes of the input solute became small.

The fractions of the input solute transported up and down are plotted in figure 6 for all the runs in figure 3 plus the extra runs, with the density ratio scale halved to accommodate the extended range. The same symbols are used as before to plot the results obtained using the inversion, and extra smaller dots have been added to show the corresponding measurements using the direct calibrations, where these are sufficiently different to be distinguished on this scale. For some of the runs in the extended range only the latter are shown, since one or more of the measured concentrations was outside the range in which the Ruddick & Shirtcliffe (1979) inversion calculation is valid. This means, of course, that the anomaly fluxes for the tank fluid could not be deduced in these cases, and no attempt will be made to extend figure 5.

The counterbuoyant flux of input fluid continues to fall off slowly, especially for the lightest sugar plumes and densest salt plumes. The reason for this can be seen in figure 7(a), a shadowgraph of an experiment using a sugar plume with $\rho_i/\rho_0 = 0.9777$, taken after 1 h at an input rate of 5 ml min^{-1} . Because of the large density difference, the plume rose nearly vertically away from the nozzle (becoming turbulent rapidly) and spread out on the free surface. It then produced a deep stratified region through the 'filling box' mechanism, with long fingers extending through the depth and an interface below it. Fluxes of both solutes continued through this interface, driven by the unstably distributed sugar, which produced strong convection in the lower layer. At the end of the experiment, about 8% of the input sugar was measured in the bottom layer.

It should be noted in passing that the counterbuoyant heat fluxes measured by McDougall (1983) for warm salty plumes in a cold fresh environment, fell rapidly to zero at a finite small density difference, without an asymptotic 'tail'. This difference is probably because he was able to measure the transports produced by plume splitting in an effectively infinite environment, before the interfacial transports or the filling box process became significant. Since both transport mechanisms inevitably contribute to the measurements presented in this paper, it does not seem appropriate to make a detailed comparison with McDougall's results.

Clearly, for the larger density differences, the interfacial vertical transport mechanism dominates over that due to the separating plumes formed with smaller density differences. The long asymptotic tail for the upward flux with dense salt plumes can also be attributed to a bottom layer stratified in the finger sense. Figure 6 shows that the upward flux of salt produced by dense salt plumes lies well above the upward flux of sugar for dense sugar plumes, implying that finger interfaces continue to give larger solute fluxes than diffusive interfaces as the density difference becomes large. As pointed out by TV, however, the coupled transports across finger interfaces lead to a smaller net density change, since the fluxes of salt and sugar are more nearly compensating than they are for diffusive interfaces.

3.5. *The effect of nonlinear density changes – cabbeling*

Another feature of the results set out in table 1 has not yet been addressed. There is a large asymmetry in the upward and downward density changes at an input density ratio of 1 – the lower layer became denser while the upper layer changed hardly at all. McDougall (personal communication) has suggested that cabbeling, i.e. the nonlinear increase of density that can occur on mixing, might be important in explaining this behaviour. He developed the argument in terms of the heat–salt case, but it can easily be adapted to apply to the sugar/salt analogue system. Suppose that there is a 50:50 mixing between the plume fluid (assumed to be undiluted initially) and the tank fluid, and that both the sugar and salt solutions have a density difference from fresh water of 0.1000 g cm^{-3} , close to the densities used in the experiments. Using the Ruddick & Shirtcliffe (1979) expressions for the density of sugar–salt mixtures, the increase in density on mixing (the cabbeling effect) is found to be $0.00055 \text{ g cm}^{-3}$. This is about one-fifth of the decrease in density of a sugar plume (0.0027 g cm^{-3}) at which there are equal upward and downward transports of sugar in figure 3. Cabbeling increases the density of the plume in both cases, but it acts on a sugar plume in the sense which reinforces the direction of motion (downwards) produced by differential diffusion across its boundary, but in the opposite sense to diffusion for a salt plume. This process could explain the much smaller density changes measured in the upper

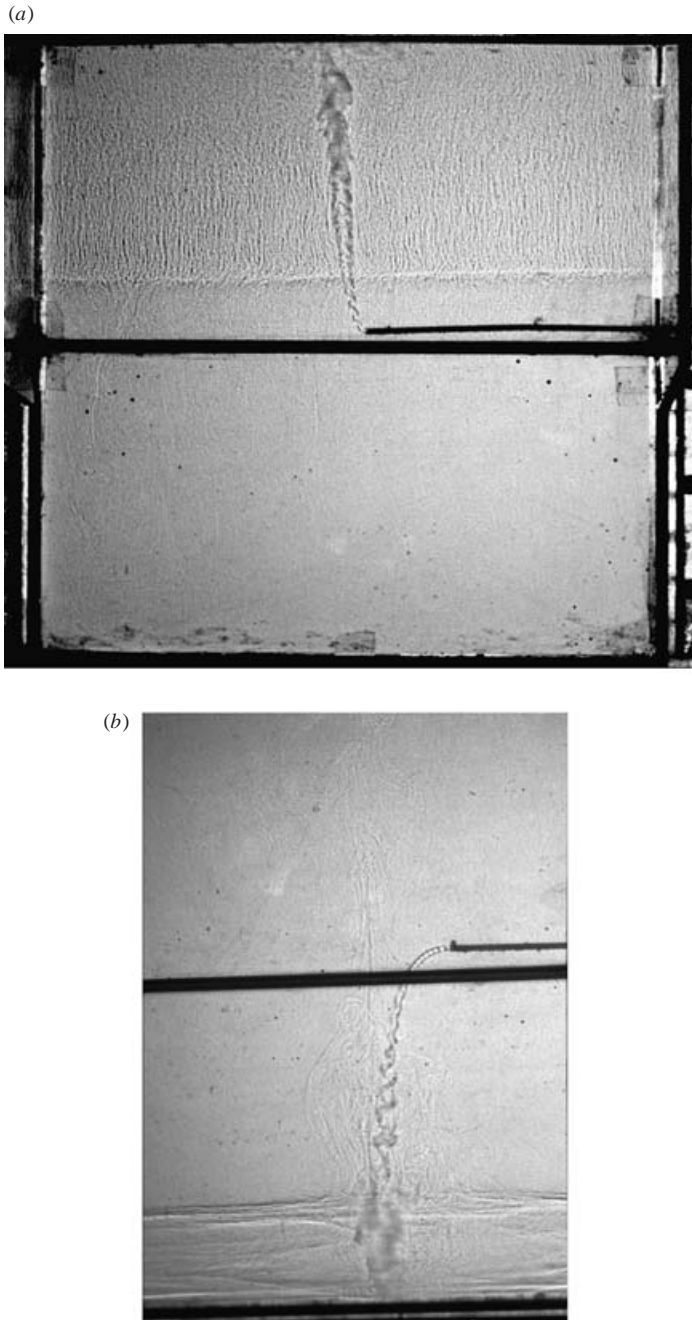


FIGURE 7. Two photographs showing the formation of layers bounded by sharp interfaces; transports across these account for the asymptotic behaviour of the fluxes. (a) A light sugar plume with $\rho_i/\rho_0 = 0.9777$ flowing into salt solution at a rate of 5 ml min^{-1} ; photograph taken after 1 h. The plume has risen nearly vertically to the surface and formed a deep layer filled with sugar/salt fingers. About 8% of the input sugar was measured in the bottom layer. (b) A dense sugar plume with $\rho_i/\rho_0 = 1.0011$ flowing into salt solution at 5 ml min^{-1} . The flow was strongly downwards and a diffusive interface formed above a growing layer; even at this small density difference, the upward flux of sugar was only about 25% of the input.

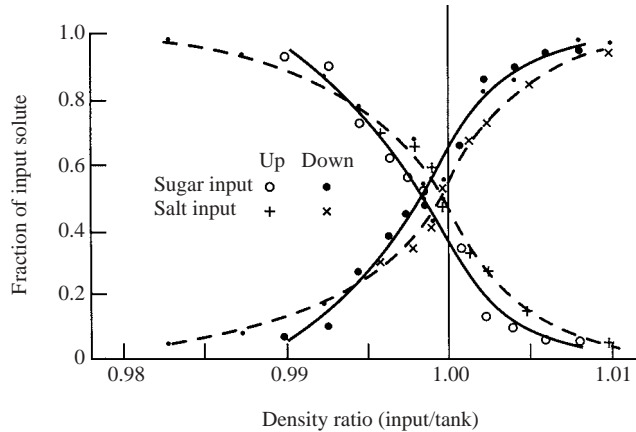


FIGURE 8. The fraction of input solute transported up and down, as a function of density ratio, for all experiments with an input rate of 20 ml min^{-1} (cf. figure 6 for the lower flow rate).

layer, but its magnitude does not seem sufficient to explain the major differences in vertical transports produced by sugar and salt plumes.

4. Experiments using an inflow rate of 20 ml min^{-1}

Two further series of experiments were carried out, to investigate the behaviour of sugar and salt plumes using a second constant higher flow rate, and to compare them with the above measurements. The results will be presented and discussed more briefly, covering the ground discussed in the previous section but without giving all the corresponding plots.

Figure 8 shows the results for the fractional vertical transports of the plume solute, for all experiments with a flow rate of 20 ml min^{-1} . This corresponds to figure 6 with the compressed horizontal scale, and it includes points, indicated by the smaller dots, which were obtained using the direct calibrations, but for which the inversion was not straightforward. The equivalent of figure 3, which was used to give a clearer presentation of the measurements at the smallest density differences for the lower flow rate, has been omitted. Again the upward and downward transports of sugar become equal only when a sugar plume is significantly lighter than the tank salt, and the salt transports are much more nearly equal at a density ratio of 1.

The fractions of the input solute transported in the counterbuoyancy sense, corresponding to figure 4, are shown in figure 9. Again, there is a big contrast between the two cases; light sugar plumes produce a much larger counterbuoyant transport than do dense sugar plumes. The counterbuoyant transports for salt plumes are more nearly equal, though in the high flow rate case, the lighter salt plumes have the higher counterbuoyant solute flux for a given density difference; note too that the curve for the lighter salt input crosses and continues above that for the light sugar input. The transports fall off much more rapidly with increasing density difference than they do in figure 4.

The fractional fluxes of the tank solute anomalies are shown in figure 10, corresponding to figure 5. Note again that because the measured values of sugar concentration lay outside the range considered by Ruddick & Shirtcliffe (1979), the redistribution of the tank solute (sugar) could not be derived easily for the lightest salt plumes, i.e. the smallest density ratios. That range would probably include a crossover

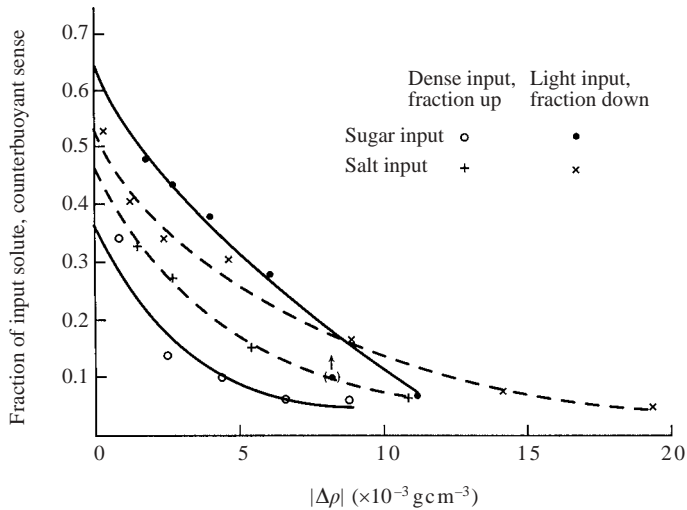


FIGURE 9. The counterbuoyant fluxes of salt and sugar at an input rate of 20 ml min^{-1} . The transports fall off more rapidly with increasing density difference than they do at the lower flow rate (cf. figure 4).

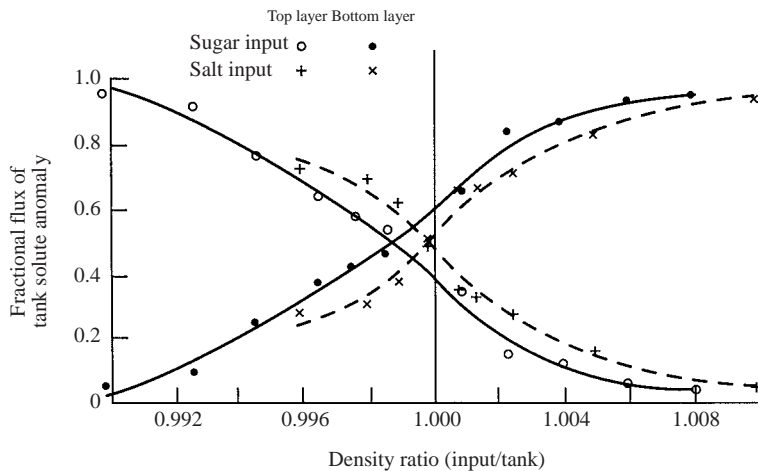


FIGURE 10. The fractions of tank solute anomalies (all negative) transported up and down, produced by convection set up by plumes of salt or sugar flowing at a rate of 20 ml min^{-1} into a homogeneous solution of the other property (cf. figure 5 for the low flow rate).

of the sugar and salt curves, with the fractional flux of sugar anomaly decreasing much more slowly than the salt anomaly in this range. The ratios of sugar and salt transports calculated for the experiments with small values of $\Delta\rho$ (and recorded in the second part of table 1) show the same pattern of variation as for the lower flow rate, with differences of detail only. When used with the fluxes at a density ratio of 1, taken from figure 8, they suggest that the stratification produced by the sugar plumes is again strongly diffusive, while the salt plumes lead to nearly uniform sugar and weakly stable salt distributions.

Two photographs of salt plumes flowing into sugar solution at 20 ml min^{-1} are reproduced in figure 11, to complement the previous pictures that have shown a

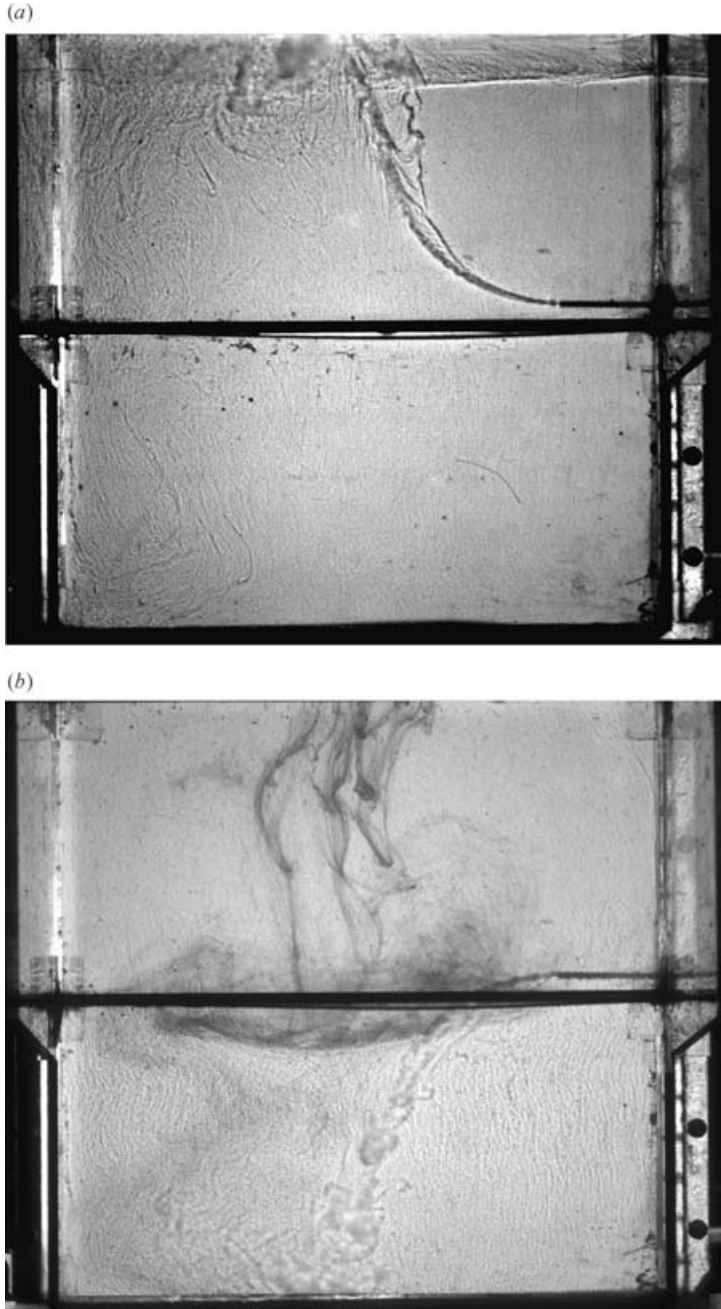


FIGURE 11. Two photographs showing salt plumes flowing into sugar solution at a rate of 20 ml min^{-1} , and the subsequent convection driven by the interfacial transports. (a) A light plume with $\rho_i/\rho_0 = 0.9921$ formed a surface layer with a diffusive interface below it. The flux through this interface was strong enough for the counterbuoyant salt curve in figures 8 and 9 to rise above the sugar curve. (b) Very dense salt plume with $\rho_i/\rho_0 = 1.00986$, forming a deep finger layer. The photograph was taken one minute after a dye streak had been added, and shortly before the experiment was terminated.

majority of sugar plumes. The light plume in figure 11(a) became turbulent rapidly and formed a layer at the surface with a diffusive interface below it, through which vigorous convection was transporting salt downwards in the counterbuoyancy sense. The density ratio for this experiment is $\rho_i/\rho_0 = 0.9921$, and at this point the salt curve in figures 8 and 9 has crossed and risen above the sugar curve. This is the only range, in all the experiments using the two flow rates, where the fractional transport of the input fluid through a diffusive interface was greater than that through a finger interface at the same density ratio.

Figure 11(b) shows a turbulent plume of salt solution with $\rho_i/\rho_0 = 1.00986$ (the densest salt plume in the series of experiments at the larger flow rate), 1 min after a streak of dye had been added at the centre, and shortly before the experiment was terminated and the partition closed. The dye marks the sharp top of a layer of fingers, through which double-diffusive convection was transporting salt upwards; the interface is just below the level of the partition, so no salt was advected directly into the upper layer. Again, as in the experiments at the lower flow rate discussed at the end of §3, the upward flux lies above the fractional counterbuoyancy transport curve for dense sugar plumes at the same density ratio.

5. Summary and discussion

The behaviour of salt plumes flowing into homogeneous sugar solution and sugar plumes into salt solution in a confined region has been investigated for a range of small density differences between tank and input fluid on either side of zero. The basic data are the densities and a second property, optical rotation for the sugar inputs and conductivity for the salt inputs, measured in the upper and lower halves of the tank (i.e. above and below the source located near mid-depth). From these, the concentrations of sugar and salt and hence the mean vertical transports of both the input and tank solutes over a relatively short time interval have been deduced. Two inflow rates, 5 ml min^{-1} and 20 ml min^{-1} , were investigated; the lower flow rate, and the mean specific gravity of about 1.100, were chosen to be the same as those used in the experiments of Turner & Veronis (2000) which provided the motivation for the work reported here.

In the TV paper, a double-diffusive system was studied as an analogue of the related two-dimensional processes in the ocean, with sources of salt (T) and sugar (S) at opposite ends of a long tank containing a homogeneous solution of either salt or sugar or a mixture of the two. Starting with equal densities of both inputs and the tank fluid, a large vertical density gradient developed, and the asymptotic vertical distributions of salt and sugar were the same (after about 100 h), regardless of the initial solution in the tank. There was, however, a substantial difference in the early evolution of the system, depending on whether the tank was filled with sugar or salt solution. The results reported here serve to explain this difference, and the measurements also shed light on the way in which the final state is achieved in the TV experiments.

Sugar and salt plumes flowing into a homogeneous tank of the other solution with a small difference in density produce systematically different vertical transports. This is evident first in the data for the fractions of the input fluid transported up and down, shown in figures 3 and 8 for experiments using the two flow rates. At a density ratio close to 1, a salt plume gives nearly equal upward and downward transports of salt, whereas a sugar plume leads to a much larger downward flux of sugar; equal upward and downward transports of sugar are measured only when the plume is much lighter

than the tank fluid. This is consistent with the asymptotic distributions of sugar and salt observed by TV, and it shows that even in the early stages of their experiments, the system was moving towards this final state. For the inflow rate used in the TV experiments, a strong diffusive stratification is set up at one end of the tank and there is a small unstable vertical salt gradient at the other (implying a weaker diffusive stratification there), so that interleaving will maintain this distribution throughout.

The tentative explanation given by Turner & Veronis (2000) to interpret their observations now seems to be incomplete, since this concentrated on the early stages of diffusion into and out of the two kinds of neutrally buoyant plume, and neglected the second stage when the transport through interfaces becomes significant. They suggested that an asymmetry between the fluxes of salt and sugar into and out of salt and sugar plumes could account entirely for the difference in behaviour; but the amount of differential diffusion of salt needed to affect the buoyancy of either salt or sugar plumes is small. The initial direction of motion will be affected by the faster diffusion of salt, and hence this will determine where the plume will deposit the bulk of its main property. A sugar plume entering a salt environment of the same density will become slightly denser and initially deposit most of the sugar at the bottom, while a salt plume in a tank of sugar solution will lose a little salt, faster than it gains sugar, so that it will become lighter and carry most of the salt upwards. The vertical gradients set up by both sugar and salt plumes immediately after they are started are indeed in the diffusive sense, but this is not the whole story.

Some of the counterbuoyant transport of the input property, when the plume does not have exactly the same density as the tank fluid, can be attributed to the direct separation of the plume near the source, as the surrounding fluid acquires buoyancy by differential diffusion, and convects in the opposite sense. This process will be enhanced as mixing occurs. However, the later vertical distributions of sugar and salt are strongly influenced also by the subsequent transports across the finger and diffusive interfaces that form as the input fluid reaches the top and bottom boundaries, as shown in the photographs of the experiments reproduced in figures 2, 7 and 11. The measurements reported here include both processes, since it was not possible to build up measurable concentrations of sugar or salt in the very short time available before the second process, the transport across double-diffusive interfaces, becomes significant.

In their discussion of the evolution of their experiments to an asymptotic state that is 'diffusive' in character, TV pointed to the very different effects of finger and diffusive interfaces. These will also be relevant during the earlier stages of the processes investigated in the present experiments. The sugar/salt flux ratio (in density terms) for diffusive interfaces is about 0.50; there is a large change in potential energy for a given salt flux since only about half the energy released from the unstable salt distribution is used to lift sugar across the interface. For a finger interface, on the other hand, the salt/sugar flux ratio is 0.90, indicating that the two transports are more nearly compensating, and there is a smaller change in potential energy. Thus, transports across diffusive interfaces are a more effective way of releasing potential energy from a double-diffusive system. The overall mean asymptotic state observed by TV did indeed seem to be that in which the maximum potential energy had been released. However, this says nothing about the magnitude of the density flux at earlier stages in the two cases. There are indications, for example in experiments on double-diffusive intrusions into a density gradient, that the density flux can be faster across a finger interface near $R_\rho = 1$, and certainly fingers can be very efficient at mixing both sugar and salt vertically, as discussed in §3.4.

The consideration of the diffusion into and out of sugar and salt plumes has also pointed to the importance of 'diffusive' type interfaces. At every sharp interface, whether these are horizontal or vertical, or at some other orientation, salt will be transferred more rapidly than sugar. The cumulative effect will be to maximize the release of potential energy which, as discussed above, cannot be accomplished primarily by salt fingering. The present experiments cover only a small fraction of the total time required to achieve the asymptotic state in the TV experiments. However, it is clear that the processes that are important in determining that final state are already active in the individual salt and sugar plumes, even before the horizontal interleaving motions have been set up that lead to the interaction between the two ends of the tank.

The results of these experiments using the salt–sugar analogue system are, of course, not directly applicable to the distributions of heat and salt in the ocean; but the principles are the same, and further investigations of the heat–salt system could be instructive. For example, the present results suggest that the behaviour of hot salty or cold fresh intrusions could be different, and not just antisymmetric as is commonly supposed. In the oceanic case the effect of cabbeling may also be more important than it is for the salt–sugar analogue, and this too should be assessed more carefully.

Correspondence and later discussions with Trevor McDougall have played a vital role in increasing my understanding of these experimental results and the most appropriate way to analyse them. He and Ross Kerr have both made many helpful comments on earlier versions of this paper, and the suggestions of the three referees have greatly improved the clarity of the final manuscript. I am grateful to Tony Beasley for his help with the experiments and to Ross Wylde-Browne and Brad Ferguson for assistance with the preparation of the photographs and figures.

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